Variation and FEM Methods Application for the Formulation of Discrete Equations of Motion in the Thermodynamic Processes

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Abstract. This paper concerns the application of variational methods for the formulation of object motion equations as well as generation heat and its transfer formulated in the updated Lagrange description. These physical phenomena occur in the processing of metals in which there are thermal, plastic and sticky deformations. The obtained incremental variation equation of the object's motion and the incremental variation of the heat conduction equation were discretized by the Finite Element Method to obtain discrete systems of equations. These equations are solved by known methods for given boundary and initial conditions. As a result, a column vector of the displacement increment and a column vector of the temperature increase in the object nodes are obtained at each time step. Examples of the use of these methods for the burnishing rolling process with electric current are shown.

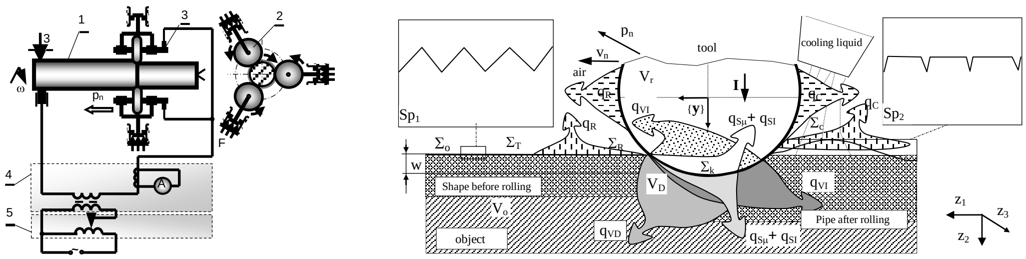
INTRODUCTION

One of the most important problems of modern manufacturing technologies is the prediction of product quality at the process design stage and during the selection of technological parameters. Typically, when analyzing these processes, it is assumed that the processes are isothermal [1-3]. This is due to the great difficulty of simultaneously taking into account both the impact of mechanical and thermal strains occurring during processing. Modern technological processes such as: cutting processes, turning processes, burnishing rolling processes, embossing , thread rolling, duplex burnishing, drawing , sliding burnishing and cutting by an abrasive single grain, from the point of view of thermodynamics of continuous bodies, are triple non-linear initial problems and boundary problems (geometrical, physical and thermal), assuming only partial knowledge of the process. The boundary conditions in the areas of contact of the tool with the object are unknown. The desire to increase the dimensional and shape accuracy of the products after these processes causes that in order to predict the quality of the product after the machining process, it is necessary to consider thermal-mechanical couplings.

In the present paper, thermodynamic technological processes were considered as triple nonlinear boundary and initial problems. The object was treated as a body in which thermal, elastic and phase deformations (in the field of reversible deformation) and viscous, plastic, thermal and phase deformations may occur (in the field of irreversible deformations). This body is briefly marked: TEF-TVPF. The formulation of discrete equations is shown in the example of a process of the burnishing rolling process with electric current (BRWC).

BUNISHING ROLLING PROCESS WITH ELECTRIC CURRENT

The burnishing rolling process with electric current process is performed with high speed heat sources which move with tool (Fig. 1).



(a) (b)

FIGURE 1. Scheme thread rolling (a): 1 – workpiece, 2 – roller, 3 – brush plate, 4 – transformer, 5 – autotransformer and diagram of the system of heat fluxes arising in characteristic volumes V and areas ∑ during rolling (b) and surface of the object and surface layers after previous treatment (Sp_1) and after rolling (Sp_2), respectively (b).

DISCRETIZED EQUATION OF THE MOTION AND DEFORMATION

A variation nonlinear equation describing motion and deformation of an object for a typical incremental step, has been spatially discretized using FEM, and a discrete system of equations for motion and deformation of the object in the burnishing process with electric current was obtained:

$$[\mathbf{M}]\cdot\{\Delta\ddot{\mathbf{r}}\}+[\mathbf{C}]\cdot\{\Delta\dot{\mathbf{r}}\}+([\mathbf{K}]+[\Delta\mathbf{K}])\cdot\{\Delta\mathbf{r}\}=\{\Delta\mathbf{R}\}+\{\Delta\mathbf{F}\}+\{\mathbf{R}\}+\{\mathbf{F}\} \tag{1}$$

where the mass matrix $[\mathbf{M}]$, damping matrix $[\mathbf{C}]$, stiffness matrix $[\mathbf{K}]$ and force internal $\{\mathbf{F}\}$ and external $\{\mathbf{R}\}$ vectors are known at time t, however the increment stiffness matrix $[\Delta\mathbf{K}]$, external incremental load vector $\{\Delta\mathbf{R}\}$, internal incremental forces vector $\{\Delta\mathbf{F}\}$, and the incremental vectors of displacement $\{\Delta\mathbf{r}\}$, velocity $\{\Delta\dot{\mathbf{r}}\}$, and acceleration $\{\Delta\ddot{\mathbf{r}}\}$ of FE assemblage at typical step time are not known. To solve of this problem is using the known integration methods.

DISCRETIZED EQUATION OF HEAT TRANSFER

Variational equation of heat transfer in the BRWC, at the typical time step, using the approximations adequate to the finite element method we obtain the discretized equation of heat transfer equilibrium in the global coordinate (the non-stationary heat transfer):

$$[\mathbf{C}]\{\Delta\dot{\mathbf{\Theta}}\}+([\mathbf{K}^{\mathrm{K}}]+[\mathbf{K}^{\mathrm{C}}]+[\mathbf{K}^{\mathrm{R}}]+[\mathbf{K}^{\mathrm{IV}}])\{\Delta\mathbf{\Theta}\}=\{\Delta\mathbf{Q}\}+\{\Delta\mathbf{Q}^{\mathrm{I}}\} \tag{2}$$

where $[\mathbf{C}]$ and $[\mathbf{K}^{\mathrm{K}}]$, $[\mathbf{K}^{\mathrm{C}}]$, $[\mathbf{K}^{\mathrm{R}}]$, $[\mathbf{K}^{\mathrm{IV}}]$ are the heat capacities, conductivity, convection and radiation matrices and total nodal point conditions of IV gender, $\{\Delta\mathbf{Q}\}$ is the nodal point increment heat flow input vector, $\{\Delta\mathbf{Q}^{\mathrm{I}}\}$ is the vector of nodal point of the boundary conditions of I gender.

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